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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

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A Dexterity Measure for the Kinematic Control of Robot Manipulator with Redundancy

Pyung H. Chang

Abstract. We have derived a new performance measure, product of minors of the Jacobian matrix, that tells how far kinematically redundant manipulators are from singularity. It was demonstrated that previously used performance measures, namely condition number and manipulability measure allowed to change configurations, causing repeatability problems and discontinuity effects. The new measure, on the other hand, assures that the arm solution remains in the same configuration, thus effectively preventing these problems.

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1 Introduction

The benefits of using a quantitative measure in engineering systems are well known. More specifically, a quantitative measure provides us with a rational basis upon which we can, without having to rely on experience and intuition alone, analyze, design, and control the systems as follows:

- one can evaluate the performance of a given system and analyze the system, by estimating this measure; or

- one can design a system that achieves the performance in a certain degree, by maximizing(or minimizing) this measure; or

- one can control, on the on-line basis, a given system to achieve it, by maximizing the measure at each moment.

In the robotic system, also, various performance measures have been incorporated to quantify desired performance features such as obstacle avoidance(Yoshikawa,1984; Maciejewski,1985; Espiau,1985; Khatib,1986), torque minimization(Hollerbach,1985), kinetic energy minimization(Whitney,1972), and constraining the joint variables within their physical limits(Liegeois,1977).

Another feature, in addition to these performance features, is to achieve *dexterous manipulation*. This performance characteristics, however, is, in fact, quite ambiguous unless the concept of *dexterity* is more precisely defined. One concept of dexterity was specified by the amount of volume in the workspace, within which the end effector can have any orientation(Vijaykumar, Tsai, and Waldron, 1985): the larger the volume, the more dexterous.

Another concept of dexterity, suggested by Yoshikawa(1985a), is the *easiness*, due to better dynamic characteristics, of changing the position and orientation

of the end effector. Another concept of dexterity proposed by Klein(1984,1987) appeared to mean (1)the goodness of a linear system of differential relationships, as indicated by either the determinant or condition number of the Jacobian matrix; or (2)a natural appearance resulting from evenly distributed joint angles, represented by summing the squares of the deviation of actuators displacements from their midpoints.

According to this definition, the *least* dexterous manipulation would happen probably at a singular point; for at a singularity the condition of a linear system is at its worst, and creates awkward appearances due to lining-up or folding of links. In this sense, therefore, dexterity may be viewed as a degree of *farness* or *distance* from a singularity. In this paper, the meaning of dexterity is explicitly specified as the *distance* from singularity.

To quantitatively represent the distance, several measures have been proposed (Yoshikawa,1985a,1985b; Uchiyama,1985; Maciejewski,1985; Salisbury,1982): the determinant of the Jacobian matrix, its condition number, and a few combinations of its singular values. It is no wonder that all of them are based on the Jacobian matrix, because, only through the matrix, the instantaneous end effector movement is determined.

Determinant

In linear algebra, the determinant of a matrix has been an important measure used to test the invertibility of the matrix and its nearness to singularity. Accordingly the determinant of the Jacobian matrix has been tried for the dexterity measure for both nonredundant and redundant manipulators. For nonredundant manipulators, for instance, the determinant has been used as a measure of degeneracy for the analysis of the wrist configurations(Paul and Stevenson, 1983). For redundant manipulators, on the other hand, Yoshikawa(1984) has proposed

a measure called *manipulability*, defined as the square root of the determinant of $\mathbf{JJ}^T$. This measure is often viewed as a generalized concept of the determinant, because of the followings:

- the manipulability reduces to the regular determinant in the nonredundant case.
- the manipulability become zero, when workspace rank reduces at singularity, just as the regular determinant of a square Jacobian matrix does.
- since the singular values of $\mathbf{JJ}^T$ have the square values of those of $\mathbf{J}$, the determinant of $\mathbf{JJ}^T$ may be regarded as if it were the square of the regular determinant of a square Jacobian matrix.

Condition number

Meanwhile, since the condition number of the Jacobian matrix is another important measure that also indicates the nearness of a matrix to singularity, it has been proposed for a dexterity measure(Salisbury,1982). It is noteworthy that this measure was initially used to determine the configuration that minimizes the propagation from the torque error to the force error — equivalently, the velocity error propagation from joint space to workspace — for nonredundant manipulator.

Singular values

The determinant and the condition number can be also expressed in terms of singular values of the Jacobian matrix: the former is the product of all the singular values, the latter the ratio of the largest to the smallest singular value. Since the minimum singular value becomes zero when the matrix is singular, and approximately determines the worst limits of the two measures, the value as such was suggested as a new measure(Klein,1985). In addition to its simple expression,

the measure has a relatively clear physical meaning: it may be interpreted as the minimum responsiveness in end effector velocity due to a unit change in joint velocity(Klein,1985).

Besides, the geometric mean and harmonic mean of singular values have been proposed for the dexterity measures(Yoshikawa,1985b), which may be viewed essentially as variations of aforementioned measures.

Common features

The features common to all these measures are as the following:

- They indicate the presence of singularity: when singular, the value of these measures become zero, except for the condition number, the value of which becomes infinity.

- Their absolute values — inverse of the value in the case of the condition number — appear to represent, in one way or another, the farness or distance from singularity. That is, the larger the value, the farther is the manipulator from a singularity.

In the case of redundant manipulators, however, these measures do not explicitly indicate the successive changes in the available degrees of freedom as long as the workspace rank is preserved. For instance, suppose we have a five d.o.f. manipulator which is to move in a three-dimensional workspace, hence having two degrees of redundancy. Although the manipulator happens to lose one degree of freedom, or even two, the measures do not necessarily indicate that fact.

Because the degree of redundancy is an important constituent of the distance from a singularity, there is an obvious shortcoming for these distance measures. Furthermore, unnoticed relative differences in the distance from singular for a particular degree of redundancy. Therefore, we feel that a satisfactory dexterity

measure should not only include the feature of indicating the change in the degrees of redundancy, but also represent relative differences in a particular degree of redundancy.

Losing degrees of freedom may not in itself be a serious drawback, as long as the workspace rank is fully preserved so that the desired location of the end effector can be achieved by joint variables. Yet, what may be of more concerns are potential problems that are expected to arise — from the similar experience in the nonredundant case — when degrees of freedom are lost. More specifically, in the nonredundant case, the point where the degrees of freedom are lost — namely the singular point — is in fact the boundary of switching from one set of joint solution to another (Uchiyama,1979). Once that switching happens, the manipulator tends to *stay* in the new kind of joint configuration different from the previous kind, thus causing a type of *repeatability problem*. Besides, when the switching arises, usually there are accompanying discontinuity in motion, resulting in large joint velocities. The same problems are expected in the redundant case, since in this case too there exist multiple solutions of different kinds (Borrel,1986), whose boundaries are the points where the degree of freedom decreases. It appears, however, that these nontrivial problems tend to be veiled because of the fact that owing to the redundancy the switching can happen without causing the more serious problem, singularity. To our knowledge, there have not appeared any analysis on these problems for redundant manipulators, and any performance measures that are intended to prevent them.

The objectives of the present paper are as the following:

- to analyze the aforementioned relative distance for redundant manipulators and to derive a new distance concept;

- to derive from this concept a new performance measure that represents the dexterity of manipulators including singularity overcoming;
- to examine if the new performance measure helps avoid the repeatability problems and discontinuous motions due to switching solution types.

This performance measure is intended to be used either with on-line kinematic control methods, or for off-line design purposes.

In order to better understand the degree of redundancy and the relative differences, we first review in Section 2 the concept of singularity, and some basic knowledge about the degree of redundancy. Then we will derive a new concept of the distance from singularity for kinematically redundant cases. This concept is obtained by observing the structure of the Jacobian matrix of redundant manipulators. Then, from this concept, a new performance measure will be developed. The property of this measure will be discussed in Section 3. Besides, the new performance measure will be qualitatively compared with two existing performance measures: the manipulability measure and the condition number. In Section 4, the numerical simulations will be made with redundant manipulators to compare the effectiveness of each measure in achieving dexterous movements. In the comparison, at the same time, the repeatability problems, as well as the ability to to preserve the kind of joint solutions are to be observed. Finally some concluding remarks will be made in Section 5.

2 New Distance Concept and Performance Measure

This subsection presents reviews on two basic concepts, singularity and kinematic redundancy for better understanding the distance from singularity in the redun-

dant manipulators. Then the distance concept and its corresponding measure will be derived.

2.1 Review of Singularity and Redundancy

2.1.1 Singularity

[1] Singularities can be easily observed by examining the differential relationship, or the Jacobian equation, which is,

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\boldsymbol{\theta}} \tag{1}$$

where $\boldsymbol{\theta}$ is n-dimensional vector representing joint variables, $\mathbf{x}$ is m-dimensional vector representing end effector location.

For the nonredundant case ($n = m$), the Jacobian matrix $\mathbf{J}$ is a square matrix. When the Jacobian matrix becomes singular, a manipulator is said to be at a *singular* point. Hence at a singular point, the determinant of $\mathbf{J}$, $det(\mathbf{J})$ equals zero. This simple fact, together with the fact that the determinant is a continuous function of joint variables, provides some important insights:

1. When $det(\mathbf{J}) = 0$, the rank of $\mathbf{J}$ is reduced, and the arm loses corresponding degrees of freedom. The result is an inability to move in some directions by any combinations of small motions in the joints.

2. Thus, as an arm approaches this point, small movements in those directions require very large displacements in joint space.

3. Since the determinant is a continuous function, at singular points, its sign changes. Since the determinant is the ratio of the differential volume of

[1] Much of discussions here is based on the personal note on singularity by Professor B.K.P. Horn, the author's supervisor.

Cartesian coordinates to that of joint coordinates, the sign change in the determinant indicates a change from one kind of solution to another. In fact, just as a change of sign in a continuous function cannot occur without passing through zero, so the arm cannot change from one kind of solution to another without passing through a singularity. This property was also discussed in (Uchiyama, 1979)

4. At a singularity, two different kinds of solution become one kind; hence, the number of different solutions is reduced.

5. Items 3 and 4 can be explained in terms of Riemann sheets: multiple solutions correspond to multiple sheets each of which represents the mapping from joint angles to Cartesian coordinates; and singular points lie on the *folds*.

The first two items may explain why keeping far from a singularity is closely related to dexterity.

From the fact that the determinant becomes zero at a singularity, it functions in a sense, as an *indicator* of the presence of a singular point. A geometrical interpretation of the absolute value of the determinant is the *volume* of a parallelepiped made of n column vectors (or row vectors) of the Jacobian matrix. This interpretation, together with the fact that at a singularity the parallelepiped collapses and the volume becomes zero, is in fact the basis of the idea that the determinant is a ***measure of distance*** from a singularity.

One disadvantage of using the determinant, however, as an indicator of singularity is that, once the rank of $\mathbf{J}$ is reduced, it does not distinguish between one state of singularity and another although their remaining ranks are different; the determinants of both of them are equally zero. The more accurate indicator for this purpose would probably be the remaining rank, or degree of freedom, itself.

For redundant manipulators, the measure equivalent to $det(\mathbf{J})$ is the manipulability measure, $\sqrt{det(\mathbf{J}\mathbf{J}^T)}$. Yet, this measure, as mentioned in Section 1, cannot indicate the change in the degrees of redundancy, which will be reviewed in the following subsection.

2.1.2 Kinematic Redundancy

The degree of redundancy r is formally defined as

$$r = n - m \tag{2}$$

where n is the degree of freedom, and m the rank of workspace. In linear algebra (Strang, 1980), the degree of redundancy corresponds to the dimension of null space of the Jacobian matrix, the degree of freedom to the dimension of its column space, and the workspace rank to the dimension of its row space. In other words, the degree of redundancy is the maximum number of linearly independent vectors in the null space, $\mathbf{e}_i$, defined as

$$\mathbf{J}\mathbf{e}_i = \mathbf{0} \tag{3}$$

But, we find that this definition is not sufficient for describing the concept of *redundancy*.

For instance, consider the following Jacobian matrix representing a three degree of freedom planar redundant manipulator consisting of three two-dimensional column vectors, J^1, J^2, and J^3

$$\mathbf{J} = (J^1 \quad J^2 \quad J^3).$$

If the second and the third links line up, that is $J^3 = cJ^2$ with c any nonzero constant, we know we do not have any redundancy left. However, the null space

vector that satisfies (3) is obtained as

$$\mathbf{e} = (0 \quad c \quad -1),$$

which is a nonzero vector. Thus, according to the definition, the degree of redundancy is one, whereas the observation indicates there is no redundancy. This discrepancy can be resolved if we modify the meaning of n in (2), as the *available* degrees of freedom. But, as mentioned in Section 1, it turns out that this modified definition is still inadequate to describe the relative differences in the distance within the same degree of redundancy.

2.2 A New Concept of Distance from Singularity in Redundant Case

In this subsection, we will observe some Jacobian matrices of kinematically redundant manipulators, and identify relative differences in the distance from singularity. On the basis of the observations, we will propose a new definition of the distance from a singularity.

Again, the kinematic equation of a kinematically redundant manipulator is generally given as follows:

$$\mathbf{x} = \mathbf{f}(\boldsymbol{\theta})$$

where $\mathbf{x} \in \Re^m$, and $\boldsymbol{\theta} \in \Re^n$ with with $m < n$. Then, the Jacobian matrix from the equation, which is given as $\mathbf{J} \in \Re^{m \times n}$, may be denoted in general as,

$$\mathbf{J} = [J^1, J^2, \ldots, J^m, J^{m+1}, \ldots, J^n]$$

where J^k is k-th column vector. If m linearly independent vectors are chosen, without loss of generality, as the first m column vectors of $\mathbf{J}$, then, from linear algebra, the remaining $n-m$ vectors $J^{m+1}, \ldots, J^n$ are linear combinations of $J^1, \ldots, J^m$ (Strang, 1980).

Observations show that *the number* of these m vectors that are included in the linear combination for each of the remaining $n - m$ vectors determines *how far* from a singularity a given configuration of the manipulator is at the moment.

To illustrate the point, let us select a manipulator with one degree of redundancy, i.e., $n = m + 1$, where

$$\mathbf{J} = [J^1, J^2, \ldots, J^m, J^{m+1}]$$

Consider the following three cases of linear combinations for J^{m-1}:

1. $J^{m+1} = a_1 J^1$

2. $J^{m+1} = a_1 J^1 + a_2 J^2$

3. $J^{m-1} = a_1 J^1 + a_2 J^2 + \ldots + a_m J^m$

where a_i's, $(i = 1, 2, \ldots, m)$ are arbitrary nonzero constants. What are then the differences among these cases?

According to the formal definition in (2), the degree of redundancy for each of the three cases are is one. Alternatively, if the modified definition is used, then the manipulator in Case 1 has no redundancy, whereas both those in Cases 2 and 3 have one degree of redundancy. However, a careful observation reveals that there still exists another difference in the distance from a singularity between Cases 2 and 3. The differences among the three cases may be explained as follows:

1. In Case 1, the manipulator gets into a singularity, reducing its rank ($< m$), if *any* two of the first m column vectors happen to line up.

2. In Case 2, singularity arises if *any* two, except for J^1 and J^2, of the m column vectors line up.

3. In Case 3, the Jacobian matrix preserves its rank ($= m$), although *any* two of the column vectors happen to line up.

In other words, the *chance* for the manipulator to get into a singularity decreases by degrees, as the number of linearly independent vectors to be included in the combination increases. These differences in chances of getting into singularity determine the *relative differences*, for system with the same degrees of redundancy, in the distance from singularity. Meanwhile, the number of $J^1, J^2, \ldots, J^m$ that appear in each of $J^{m+1}, \ldots, J^n$ uniquely determines the number of distinct combinations of m linearly independent column vectors, or the number of distinct submatrices of rank m in the Jacobian matrix. Hence, this number of submatrices also represents the margin from singularity; as the number increases, the system is less likely to become singular. Of course, this number reduces as the number of column vectors which line up increases. While these two measures are equivalent, determining the number of submatrices would be much easier than selecting redundant vectors in the set of m vectors. Note, at the same time, that the observation is not confined to this particular example of a one degree of redundancy case, but evidently true for general cases, where the degree of redundancy is more than one.

As another example, consider the following five jointed robot having a three dimensional workspace and thus two degrees of redundancy, where the Jacobian matrix is given as,

$$\mathbf{J} = [J^1 J^2 J^3 J^4 J^5]$$

where J^i's are again the three dimensional column vectors. If J^1, J^2, and J^3 are selected as linearly independent vectors, then J^4 and J^5 are, in general, represented as

$$J^4 = c_1 J^1 + c_2 J^2 + c_3 J^3$$

$$J^5 = d_1 J^1 + d_2 J^2 + d_3 J^3$$

Depending on *how many* and *which* of c_i's and d_i's are zero, we have different numbers and combinations of linearly independent vectors appearing in J^4 and J^5. At the same time, this number and combination of vectors determine the number of submatrices of rank 3 in the Jacobian matrix. The Table 1 shows the relationship between the number of submatrices and the number (and the combination) of linearly independent vectors.

It is noteworthy that the number of submatrices *successively* reduces from the maximum, 10, to the minimum, 1, depending on the number and combination of linearly independent vectors. Again, even within the same degree of redundancy, there are different number of submatrices. Clearly this number of submatrices differentiates the relative distance from singularity.

In addition, note that the absolute value of the determinant of each submatrix, called a *minor*, represents the distance from its own degenerating state. Therefore, a measure of the overall distance from a singularity should consider the value of each minor of the Jacobian matrix. In other words, in addition to the *number* of submatrices of rank m, the chance of singularity is even less, as *each submatrix* is farther from a singularity, a larger absolute value of the minor.

The above observations directly lead to a definition of *distance* from singularity as follows:

Definition

The distance from singularity is represented as *the number* of distinct nonsingular submatrices of rank m and the *magnitude* of determinant of each submatrix, or the magnitude of each minor of the Jacobian matrix.

2.3 The Derivation of A New Performance Measure

Based on the distance concept developed, we will derive a performance measure for the purposes of kinematic control and manipulator design More specifically, the following objectives are simultaneously to be met in order to achieve the desired performance:

- to keep *the number* of distinct nonsingular submatrices of rank m as large as possible;
- to make *the magnitude* of each minor as large as possible.

As an index that explicitly represents these objectives, we propose the following measure:

$$H = \left| \prod_{i}^{p} \Delta_i \right|^{1/p} \tag{4}$$

where the Δ_i's for $i = 1, 2, \ldots, p$, with $p = nCm$, are minors of rank m of the Jacobian matrix. Clearly, this measure contains in its expression the two elements of the distance, the *number* and the *magnitude* of distinct minors, in such a way that both objectives are automatically achieved as it increases. To be more specific, since the measure has nonzero values only if all of the minors are nonzero, keeping it greater than zero guarantees the maximum *number* of distinct submatrices. At the same time, since the measure cannot have a large value unless each minor is large, increasing the measure tends to increase the *magnitude* of each, as a whole. Furthermore, since the measure is a product, it becomes smaller if the minors have uneven values. Therefore, this prevents any minor from being particularly large at the cost of forcing others to be too small.

In (4), the exponent $1/p$ is primarily used so that, when $n = m$, the measure might reduce to the absolute value of the determinant. We find a similar

treatment, in (Yoshikawa, 1984), where the manipulability measure is defined by applying the exponent of 1/2 to $det(\mathbf{J}\mathbf{J}^T)$. Of course, if exponents are used, then physical interpretations for the measure become different. This will be considered in the next section. Besides, the use of an exponent, when the measure is used in the null space of the Resolved Motion Method, results in a different time response of convergence toward the optimal joint configuration. Except for these differences, the essential characteristics are not changed.

3 The Properties of The New Measure and Its Relationship with Other Measures

3.1 The Propertis of The New Measure

Examining the new measure, we find the following important properties:

- When $m = n$, i.e., for nonredundant manipulators, the measure reduces to

$$H = |det(\mathbf{J})|$$

  which is the same as that proposed by (Paul and Stevenson, 1983). This measure may be *conceptually interpreted* as the *volume* of a parallelepiped in m-dimensional space, the edges of which come from the rows — or equivalently columns — of the Jacobian matrix, **J**.

- When $n > m$, the measure represents the *geometric mean* of the volumes of p parallelepipeds made of each combination of m column vectors out of n.

- The points where $\Delta_i = 0$ determine the boundary between one *kind* of joint configuration (or solution) and another kind. These points are also the

points where some of the column vectors in Δ_i are linear combinations of the remaining ones in Δ_i, and thereby causing the minor to become zero.

Note that the last property may be considered an extension of the nonredundant case in Section 2 to the redundant case where the points satisfying $det(\mathbf{J}) = 0$ determines boundaries. This property, in fact, was used by Borrel and Liegeois (Borrel, 1986) to determine the boundaries of different kinds of joint solutions. These boundaries then divide the joint space into subsets, called *aspects*, each of which consists of one kind of joint solution or configuration.

In addition to determining aspects, we can use this property to *make* the joint configuration stay within a preferred aspect. More specifically, by keeping Δ_i nonzero, we preserve the kind of joint solutions for redundant manipulators.

Then why do we need to make the joint configuration stay within an aspect? The reasons have been mentioned in Section 1 as follows:

- The switching of aspects can cause a certain type of repeatability problem.
- Discontinuities in motion and awkward configurations may accompany the switching.

Clearly, now that maximizing the performance measure directly prevents Δ_i from becoming zero, it immediately addresses these problems. In other words, by virtue of its property, the new performance measure is expected to help solve the problems.

3.2 Relationship with Other Measures

In this subsection, we investigate the relationship between the proposed measure and the two others: first the manipulability measure and then the condition number.

3.2.1 the Relationship to the Manipulability Measure

By the fact that both measures represent the distance from a singularity, the new measure and the manipulability measure are loosely related. What is the precise relationship between the two? The following theorem answers this question:

Theorem 1 *For any matrix,* $\mathbf{J} \in \Re^{m \times n}$, *with* $m < n$,

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{i=1}^{p} \Delta_i{}^2$$

where Δ_i*'s,* $i = 1, 2, .., p$, *with* $p = nCm$, *are again minors of rank* m *of the matrix* $\mathbf{J}$.

The proof for the theorem is in Appendix 1. Since the manipulability measure, H_1, is defined as

$$H_1 = \sqrt{det(\mathbf{J}\mathbf{J}^T)}$$

it is expressed in terms of minors as follows:

$$H_1 = \sqrt{\sum_{i=1}^{p} \Delta_i{}^2} \tag{5}$$

whereas the new measure expressed in (4) is again,

$$H = \left| \prod_{i}^{p} \Delta_i \right|^{1/p}$$

Comparing the two measures, we note the following differences:

1. Geometrically, the manipulability measure may be interpreted either as the Euclidean norm of the vector representing the present state in the $\Delta_1 - \Delta_2 - \cdots - \Delta_p$ coordinates system, or as the distance from the origin to the present state in that coordinate system. In contrast, the new measure represents the radius of sphere whose volume is equal to that of the hyper-hexahedron made from the Δ_i, $i = 1, 2, \cdots, p$ coordinates in the same coordinate system.

2. As mentioned in Section 3, the new measure cannot have a large value if the values of the Δ_i's are uneven; whereas the other measure can still have a large value if only *some* of dominant minors have large values. The manipulability measure can have, in the extreme, some zero minors, as long as the workspace rank is preserved.

   Hence the new measure tends to give more balanced minors than the manipulability measure, not to mention the fact that it prevents minors from being zero, thus directly controlling the switching of aspects. In contrast, the manipulability measure does not have an immediate effect on the switching of aspects.

3. Note that the manipulability can be also expressed as (Yoshikawa, 1985)

$$H_1 = \prod_{k}^{m} \sigma_k$$

   where σ_k is the $k-th$ singular value of $\mathbf{JJ}^T$.

   This expression shows that the measure has a similar form to the new measure in that it is a product; the difference is that the manipulability measure is the product of the singular values representing the workspace, whereas the product of minors in the new measure represents the joint space.

   This difference implies, in a sense, that the former concentrates on preserving the workspace rank while the latter concentrates on degrees of freedom of joint space. Since keeping as many degrees of freedom as possible in joint space automatically preserves the workspace rank, the latter has the more sufficient yet the stricter requirements.

To illustrate the second difference, let us consider a three degree of freedom redundant manipulator as shown in Figure 1, which is to locate the end effector at a certain $x-y$ position.

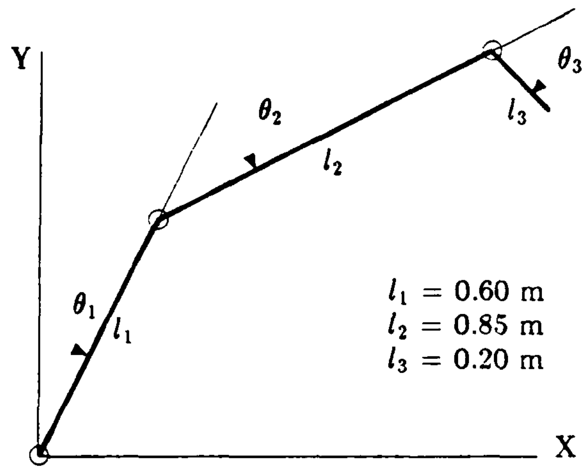
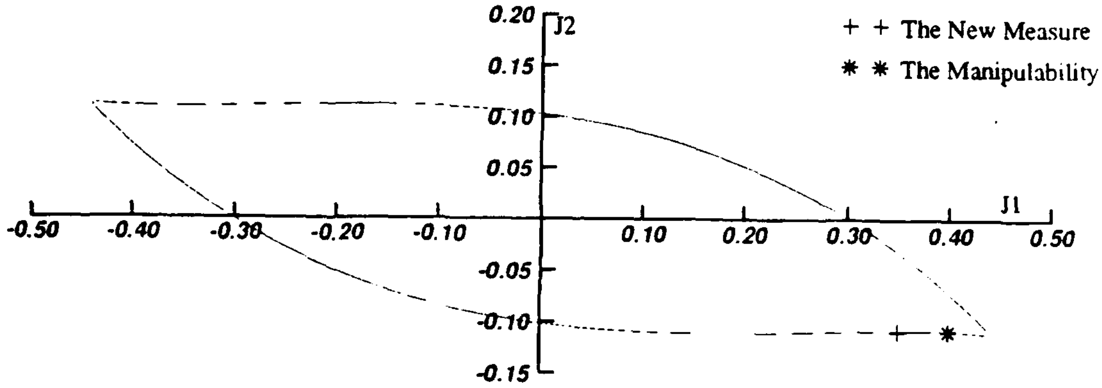


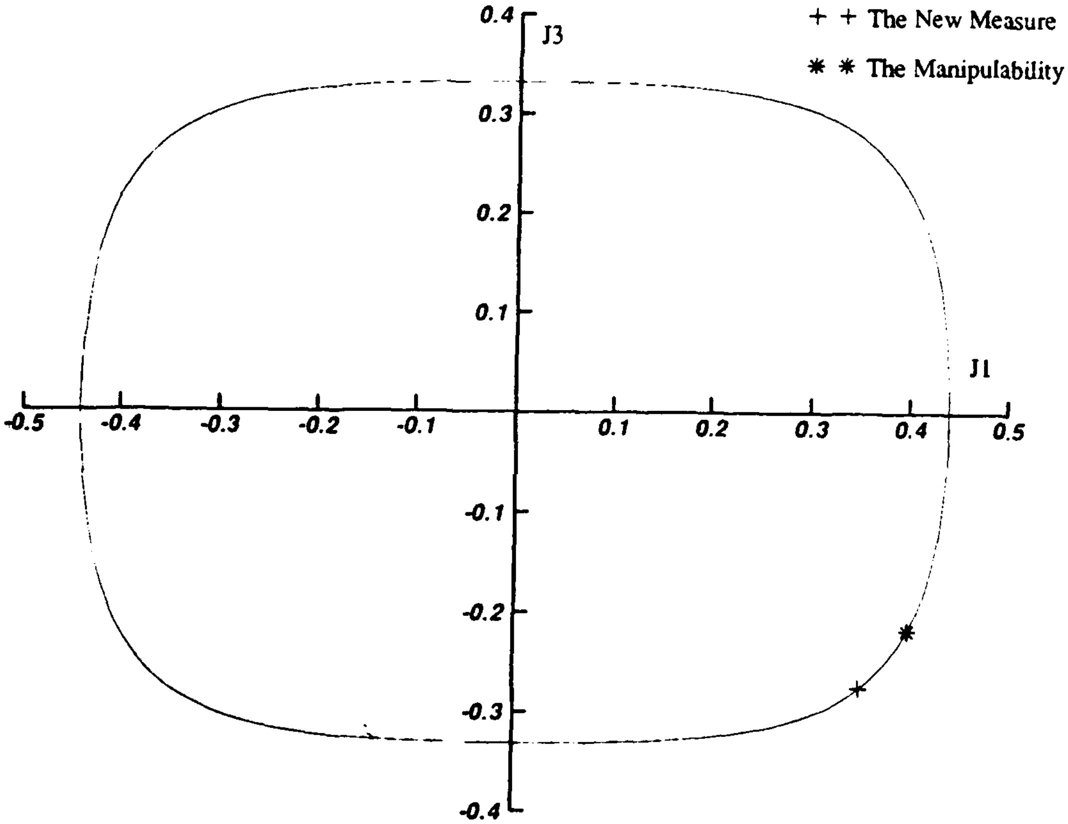
Figure 1: The Schematic Diagram Of The Redundant Manipulator

At the end effector location, there exists an infinite number of configurations or sets of joint values for a given position. Each configuration is represented by a distinct Jacobian matrix and thus a distinct set of minors.

For a given end effector location, these sets of minors accordingly determine a curve in the $\Delta_1 - \Delta_2 - \Delta_3$ coordinate system, as shown in Figure 2. Applying the inverse kinematic method presented in (Chang, 1986), when the end effector is located at $x = 0.2m \quad y = 0m$, we can obtain two sets of joint values that maximizes the two measures. Then their corresponding sets of minors are plotted in the same curve in Figure 2. These plots confirm the predicted tendency: the new performance measure gives somewhat more balanced minor values than the manipulability measure. The minor balance changes with the end effector location; this improvement is more noticeable as the tip moves toward the outer or inner workspace limits.



Cross Plot of Minor Trajectory: J1 vs. J2



Cross Plot of Minor Trajectory: J1 vs. J3

Figure 2: Trajectories of minors, $J1, J2$, and $J3$, under the constraint of kinematic equation, when the tip is at $x = 0.2, y = 0$; and two optimal configurations with relative to the two performance measure; the three dimensional trajectory is here represented with $J1$ vs. $J2$ and $J1$ vs. $J3$ trajectories.

3.3 Relationship with the condition number

The relationship with the condition number, however, is not so clear as that with the manipulability measure, because of the difficulty in deriving such a pair of simple expressions as (4) and (5). As is well known, the condition number H_2 is defined by

$$H_2 = \frac{\sigma_{max}}{\sigma_{min}} \tag{6}$$

where σ_{min} and σ_{max} are minimum and maximum values of singular values, respectively. The singular values correspond to the workspace rank: a nonzero value of σ_{min} guarantees workspace rank. Therefore, minimizing the condition number, in effect, results in maximizing σ_{min}. This measure thus tends to weight preservation of rank without weighting what happens within the redundant degrees of freedom.

4 Numerical Simulations

In this section, the new measure is quantitatively compared with the two other measures. To this end, some numerical experiments were carried out for the case of a three degrees of freedom planar manipulator. We will examine through the experiments:

1. if the new measure can help achieve the desired performance, avoiding singularities, if used for kinematic control;
2. if the measure can preserve *the aspect* (or the kind of joint configurations) and how this relates to the repeatability problem;
3. what other effects the transition of aspects brings about.

To examine the first point (the ability to overcome singularity), simulations are performed for when the manipulator has a nearly singular configuration; and when the end effector touches the base. To examine the second and third points. the end effector is made to radially reciprocate between the base and outer limits of the workspace.

The inverse kinematic method (Chang, 1986) or the resolved motion method (Liegeois, 1977) were alternately used for the kinematic control in the experiments. More specifically, the inverse kinematic method obtains the joint variables, $\boldsymbol{\theta}$, by numerically solving the following system of nonlinear equations:

$$\left\{ \begin{array}{ll} \mathbf{x} & = \mathbf{f}(\boldsymbol{\theta}) \\ \mathbf{Zh} & = \mathbf{0} \end{array} \right\} \tag{7}$$

where the upper equations are the kinematic equations, and the lower equations the optimizing equations, where $\mathbf{Z}$ is the null space matrix defined as

$$\mathbf{Z} = \left[\mathbf{J_{n-m}}\mathbf{J_m}^{-1} : -\mathbf{I_{n-m}}\right], \tag{8}$$

while $\mathbf{h}$ is the gradient of performance measure functions as

$$\begin{array}{ll} \mathbf{h} & = (h_1, h_2, \ldots, h_n)^T \\ h_i & = \frac{\partial H}{\partial \theta_i}, \quad (i = 1, 2, \ldots, n) \end{array} \tag{9}$$

Meanwhile the resolved motion method is to solve

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^+\dot{\mathbf{x}} + \alpha(\mathbf{I} - \mathbf{J}^+\mathbf{J})\mathbf{h}, \tag{10}$$

where α is a gain constant, $\mathbf{I}$ the n-dimensional identity matrix, and $\mathbf{J}^+$ the matrix known as Moore-Penrose pseudoinverse defined as

$$\mathbf{J}^+ = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}. \tag{11}$$

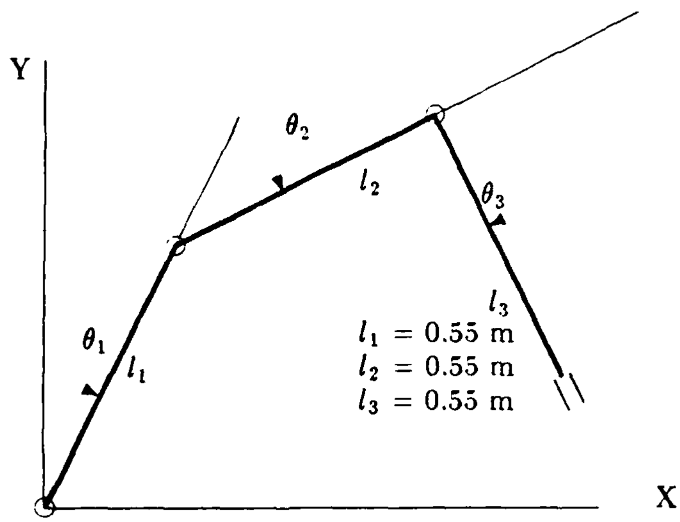


Figure 3: The Schematic Diagram Of The Redundant Manipulator With Links of Equal Length

4.1 Overcoming Singularity

The two experiments examining the ability of overcoming singularity are made with a manipulator that has three revolute joints with equal lengths of $0.55m$, as shown in Figure 3.

4.1.1 Escaping from a nearly singular configuration

In the first experiment, starting from a nearly singular configuration of $\boldsymbol{\theta} = (-90^o, 179.5^o, 0^o)^T$, the manipulator is commanded to have self-motion using each of the three performance measures, and the respective results are compared.

The resolved motion method is used, with $\alpha = 10$ in (10), to examine the change of configurations. In order to avoid a large value of null space term, the condition number was minimized by maximizing its inverse.

Table 1: The relationship between the number of linearly independent vectors in representing the remaining vectors and the number of submatrices in the Jacobian matrix.

c_1 c_2 c_3	* * *	* * *	0 0 *	0 * *	0 0 *	0 * *
d_1 d_2 d_3	* * *	0 * *	* * *	* * 0	* * 0	0 * *
No. of submatrices	10 ($5C_3$)	9	8	8	7	6
Notes:	A	B	C	D	E	F
c_1 c_2 c_3	0 0 *	0 0 *	0 0 0	0 * 0	0 0 0	0 0 0
d_1 d_2 d_3	0 * *	* 0 0	* * *	0 * 0	0 * 0	0 0 0
No. of submatrices	5	4	4	3	2	1
Notes:	G	H	I	J	K	L

* represent any nonzero value

<u>Notes:</u>

A All of c_i's and d_i's are nonzero.

B Only one among c_i's and d_i's is zero.

C Any two of either c_i's or d_i's zero.

D One of c_i's and one of d_j's are zero with $i \neq j$.

E Two of either c_i's or d_i's are zero and one of the other parts, d_j's or c_j's, is zero with $i \neq j$.

F One of c_i's and one of d_i's are zero with $i = j$.

G Two of either c_i's or d_i's are zero and one of the other parts, d_j's or c_j's, is zero with $i = j$.

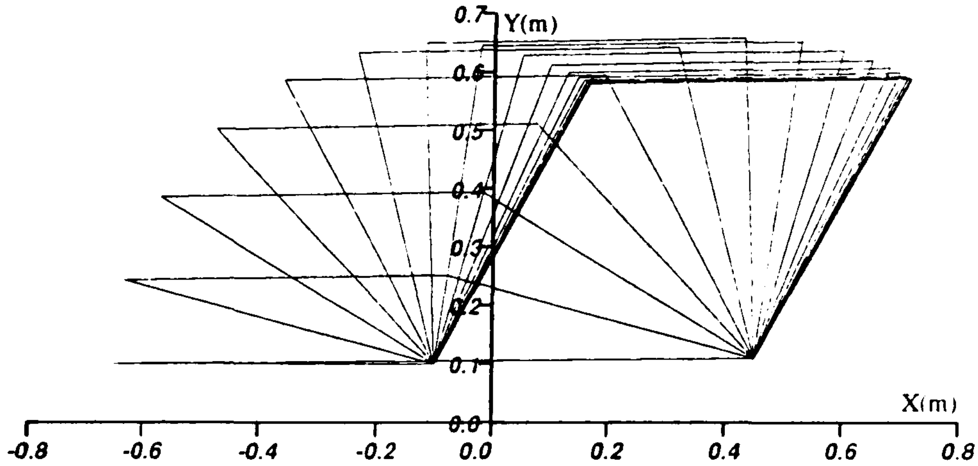
H Two of both c_i's and d_j's are zero, with one overlapping $i = j$.

I All of either c_i's or d_j's are zero.

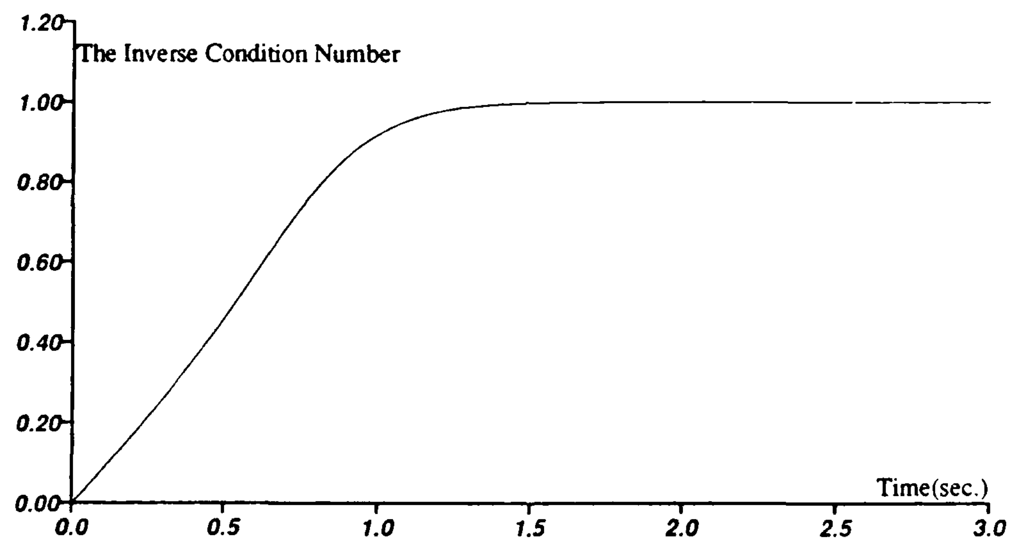
J One of both c_i's and d_j's are nonzero, with $i = j$.

K Only one of either c_i's or d_j's is nonzero.

L All of c_i's and d_j's are zero.

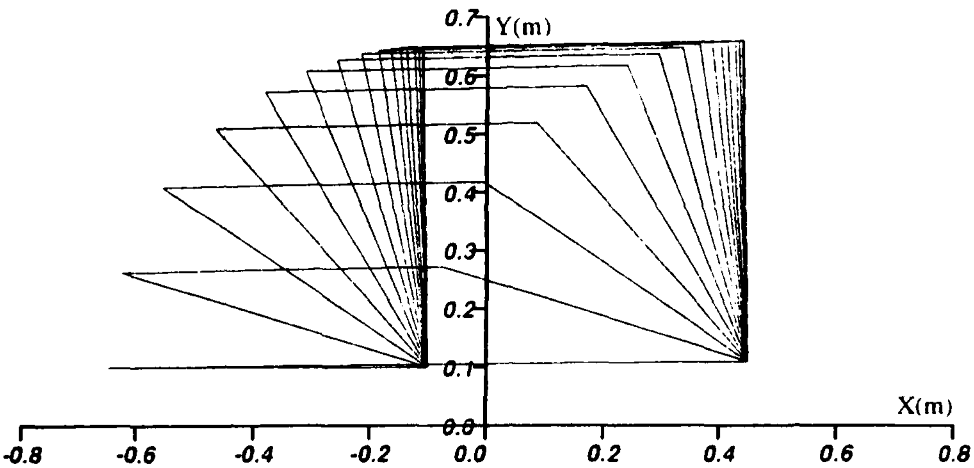


The Condition Number Is Used

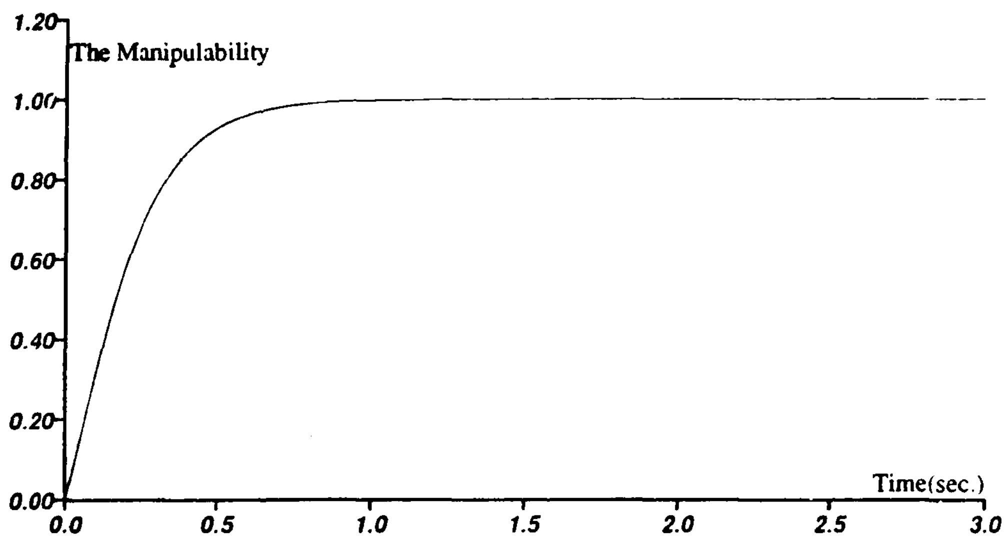


Time Response of The Inverse Condition Number

Figure 4: The Singularity Avoidance Ability When The Condition Number Is Used: The Configuration Is Changing Toward The Optimal Configuration As Time Goes On. The Link Lengths Are $l_1 = l_2 = l_3 = 0.55m$.



The Manipulability is Used



Time Response of The Manipulability

Figure 5: The Singularity Avoidance Ability When The Manipulability Measure Is Used: The Configuration Is Changing Toward The Optimal Configuration As Time Goes On. The Link Lengths Are $l_1 = l_2 = l_3 = 0.55m$.

The result of the experiment is given in Figures 4, 5, 6, where the change of configurations and the time response of each measure are shown. From this result, it is clearly demonstrated that each measure, if included in the null space term, makes the manipulator escape from the singular configuration, driving joint values toward the state where the measure has the maximum value for the tip location. The effect of including the performance measures is significant because there is no self-motion *without* the use of these measures. The speed of convergence for the condition number case is noticeably slower than those of the two other measures, both of which are almost the same.

Whereas the condition number has a considerably different steady state configuration, the new measure and the manipulability measure have steady state configurations that look surprisingly similar. Yet a close inspection shows that they are slightly different. The reason for these similar configurations is as follows. The optimizing equation, $\mathbf{Zh} = \mathbf{0}$ in (7), for each measure is in general quite different from one another. Even for the manipulator in Figure 3 with such a particularly symmetric geometry, joint solutions for each measure are different. However, the above optimal condition for each measure turns out to be satisfied only at end effector locations (x, y) satisfying $x^2 + y^2 = l^2$, with a particular joint values of $\theta_2 = \theta_3 = 90^o$ or $\theta_2 = \theta_3 = -90^o$. Here l is the value of the length of the three links of the given manipulator.

4.1.2 When passing the base

Whereas the previous experiment examines the self-escaping ability, the present one examines the behavior of the manipulator when the tip touches the base, forming a closed kinematic chain — a triangle. This case is of interest because intuitively we see the self-motion is not possible, except for rigid body rotation

of the triangle with respect to the base, as long as the tip stays at that location. This intuition can be easily confirmed if the projection matrix, $\mathbf{I} - \mathbf{J}^{+}\mathbf{J}$, in the homogeneous solution term of (10), is symbolically derived. To be more specific, when the tip is located at the base, the Jacobian matrix in (1) is modified as the following symbolic form:

$$\mathbf{J} = \begin{pmatrix} 0 & j_{12} & j_{13} \\ 0 & j_{22} & j_{23} \end{pmatrix} \tag{12}$$

from which the projection matrix can be derived as,

$$\mathbf{I} - \mathbf{J}^{+}\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{13}$$

Clearly, through the projection matrix, only θ_1 is affected by the gradient of performance measures, $\mathbf{h}$, resulting in a rigid body rotation of the triangle with respect to the base, without making any other changes in the configuration.

Then, is it impossible for the manipulator to get into and out of the configuration? In other words, can we obtain the inverse kinematic solution that resolves the motion when the tip is passing the base? The answer for the question is that, although the homogeneous term becomes ineffective with the tip at the base, it is still possible for the manipulator to get into and out of the point. The reason for the answer may be analyzed as follows:

- When the tip is *approaching* the base, the homogeneous solution term, although diminishing, still exists, continuing the effort to achieve the optimal configuration, until the tip touches the base.

- When the tip is *getting out* of the base, now that the projection matrix is given as in (13). The homogeneous term does not contribute to overcoming the closed chain configuration. Yet, since the rank is still preserved, the

pseudoinverse $\mathbf{J}^+$ is available, which can be derived from (11) as

$$\mathbf{J}^+ = \begin{pmatrix} 0 & 0 \\ \dot{j}_{23} & -\dot{j}_{13} \\ -\dot{j}_{22} & \dot{j}_{12} \end{pmatrix} \tag{14}$$

Since, in this matrix, the second and the third row vectors are linearly independent, we may have a differential tip displacement in *any* direction we like in the workspace.

- Then, once the tip is apart from the base — no matter how small the distance may be — the homogeneous term immediately begins to restore its effectiveness.

To sum up, at the base, where the particular solution term is still well defined, this term drives the tip out of it, while the homogeneous term is momentarily ineffective; at the remaining region in the workspace, both terms are effective. Furthermore, between the two regions, the respective transitions of the two terms are smooth without discontinuity.

One may suspect that the overcoming ability could have come from the *inexact* tip location — the tip can be slightly off the base — due to the linearization characteristics of (10). On the other hand, the inexact Jacobian matrix may have made it possible for the tip to get out of the base, which, with the exact Jacobian matrix, might be impossible. But these are not the cases, because both the Jacobian matrix in (12) and the pseudoinverse in (14) are exact expressions defined at an exact point (the base). Rather, the ability comes from intrinsic back-up function of kinematic redundancy.

The aforementioned analysis is well confirmed in the following experiment. In the experiment, the tip is made to move along the straight line starting from $\mathbf{x} = (0.2, 0)^T$ to $(-0.2, 0)^T$, passing the base, $(0, 0)^T$, with units in meters. Together

with the tip motion, the three measures are included in the resolved motion method, which provides the exact equilibrium solution after sufficient amount of iterations (Chang, 1986), thus excluding the effect of inaccuracy in both the tip location and the Jacobian matrix.

From the result shown in Figures 7, we see that with the three measures the manipulator had no difficulty in getting into and out of the point. We may conclude that the use of a redundant manipulator seals the hole in the workspace at the origin, where, without the kinematic redundancy, a singularity is unavoidable.

In the Figure 6, one may note the smoothness of motion when the new measure is used, as compared to motions with the other measures: When the other measures are used, the motions approaching the base from $\mathbf{x} = [0.2, 0]^T$ are abrupt. in θ_1. The reason for the smoothness is not very clear right now; but we can guess that keeping minors balanced prevents the abrupt changes in the joint angles.

4.2 Preserving the aspect and its effect to the repeatability problem

In the following experiment, we examine whether the manipulator, with the new performance measure, can preserve the aspect and, compare it to the cases with other measures. The manipulator to be used for this purpose is a three degree of freedom planar manipulator with revolute joints of $l_1 = 0.6m, l_2 = 0.85m$, and $l_3 = 0.2m$ with units in meters.

In the experiment, the tip is made to reciprocate radially between the base and the outer limit, where the manipulator fully extends. The radial motion itself is not of primary concern. The tip motion is made because it is a way of scanning the workspace to examine the ability to preserve the aspect. Because of rotational symmetry, a series of configurations corresponding to the tip reciprocating in

one radial direction represent configurations in all of the other directions, thus covering the whole workspace. Of course, the rotational symmetry lies in the fact that the new performance measure, together with the other measures, depends on θ_2 and θ_3 only, and is independent of θ_1 — hence, one optimal configuration for a fixed tip location is symmetrical to any other location which is the same distance from the base.

Of the two kinematic control methods, the inverse kinematic method is the more convenient one for obtaining the equilibrium states. Therefore it is mainly used together with the three measures. When applying the method, to solve for the successive joint configurations as the tip reciprocates, the present joint values are used as the initial conditions for the next tip location. The very first joint configuration corresponding to the starting point of the tip, by the way, are determined by obtaining the global minimum. To do this, we first determine all the local minima by providing every possible initial condition for solving the system of nonlinear equations, (7). These initial conditions, in turn, may be determined by finely tessellating the joint space, that is, the domains of joint variables, $\theta_1, \theta_2, \theta_3$. In parallel to this, all the local maxima at each tip location are obtained with the proposed method, in order to examine if the successive generations of joint values are indeed correct. In addition to the joint configurations, corresponding minor values are obtained to examine the correlation between joint configurations and minor values.

In Figures 8, 10, 12, the optimal joint configurations based on each of the performance measures and the value of each measure are plotted.

Corresponding minor values are plotted in Figures 9, 11, 13.

As shown in the figures, each performance measure has two distinct *sets* of configurations that, depending on the tip location, alternately give the global op-

timum. Hence, each of the two sets of configurations has its own corresponding performance measure curve: the one, consisting of mostly the ladder-shaped configurations, corresponds to the curve with a solid line (Configuration A); and the other, consisting of mostly 'N'-shaped configurations, to the curve with a broken line (Configuration B).

Besides, note that the mirror-image sets of Configurations A and B with respect to the x-axis are not included in the figures, since they have the same values of the corresponding measure. Of course, there are still additional sets of configurations — corresponding to local maxima instead of global maxima — that are not shown in the figures either.

What do we find from the resulting configurations? Let us examine the configurations generated by each performance measure one after another.

4.2.1 The Manipulability Measure

Figure 8 shows the two configurations and the corresponding values of the performance measure using the manipulability measure. As shown in the figure, depending on the tip location, the two configurations alternately assume larger performance measure values than the other. In the region between $x = 1.1m$ and $x = 1.6m$, however, the two configurations become identical, having the same performance measure values. Then what happens with the two configurations, when the tip is coming out of this region of the identical configuration? To answer this question, we need more careful observations as follows.

In Configurations A, the initial configuration is preserved within almost the entire workspace except for the region between the base and $x = 0.1m$. That is, except for this region, the initial shape is independent of the tip location and to direction of tip motion — toward or away from base.

In Configuration B, on the other hand, where the tip starts near the base, the initial configuration is preserved only if the tip is located within a certain distance from the base (about 1 m). Outside of this range, the configurations *shift* or *merge* into Configuration A. And once merged, configurations corresponding to subsequent tip motions stay within Configuration A, never returning to Configuration B.

Here, we observe that the manipulator has switched the aspect or the kind of joint solutions. Moreover the configuration, once switched from one aspect to another, does not return to the initial configuration: this is the source of the expected repeatability problem. Then, what happened to the minors when this switching of aspect occurred? Did they change their sign, passing through zero values? Figure 9 clearly shows that is the case: the tip location where merging happens is the place where one of the minors changes its sign.

4.2.2 The Condition Number

In the case of the condition number, the situation is even more complicated. In this case, Configurations A consists of successive joint configurations, where the tip starts from close to the outer limit of the workspace and moves toward the base, whereas Configuration B represents the movements in the opposite direction from the base. Differently from the manipulability case, *both* Configurations A and B, merge into the same configuration. Furthermore, the locations where mergings occur are different: about $x = 0.7m$ for Configuration A; and $x = 1.3m$ for Configuration B. And within $x < 0.7m$ or outer than $x > 1.3m$, the two configurations are identical, giving the same measure curve.

Hence, the repeatability problem occurs, between $x = 0.7m$ and $x = 1.3m$, when the tip reverses its direction after experiencing a merging. Again, the minor

curve in **Figure 11** shows that, when the switchings occur, signs of minors change.

4.2.3 The New Measure

When the new performance measure is used, there still exist two distinct configurations. One thing particularly noticeable is that there is no switching for both configurations; there is no repeatability problem at all. Because of no merging effects, the initial configurations are distinctly preserved showing also distinct measure curves. As expected, the minor curve in Figure 13 clearly shows that there is no sign change at all for the three minors. We see from this an obvious consequence of using a measure that has a direct control over each minor value.

Besides, we can observe, as compared to simulations using the other measures, smoother movements near the base.

4.3 Discontinuity effects

As mentioned in Section 3, when merging of configurations or switching of aspects occur, discontinuous joint motion was predicted.

To confirm the prediction, we obtained joint velocities of both configurations for each measure. Here the tip is made to move with a constant velocity of $0.1(m/sec)$. To resolve the velocity, we used the Resolved Motion Method.

Figure 14, 15, 16 show the resulting velocity curve corresponding to the configurations obtained in the previous subsection. As expected, simulations in which there is no switching (both simulations using the new measure and the Configuration A of the manipulability measure experiments), show quite smooth joint velocity trajectories as plotted in Figures 14 and 16.

For the remaining cases, where switchings occur, the velocity trajectory is generally rugged, confirming our prediction. Yet, the degrees of ruggedness for the

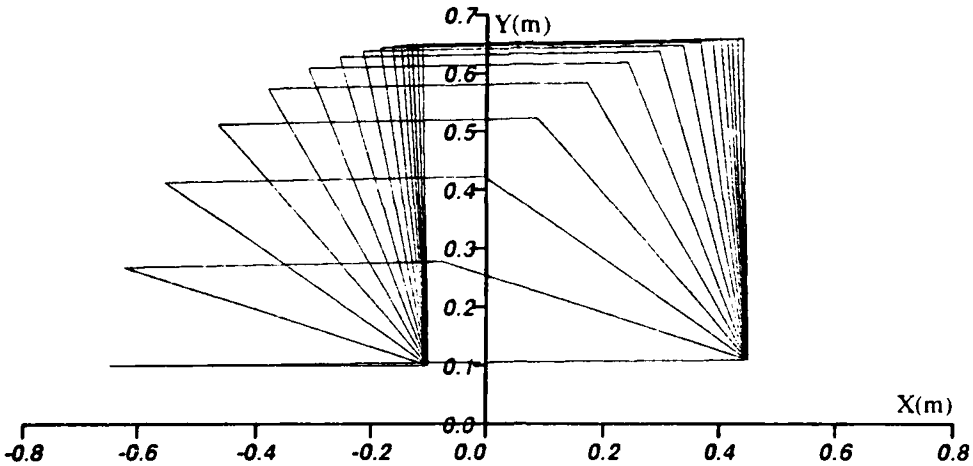
two measures are different. A careful observation reveals the following: for the manipulability measure, the merging happens through some intermediate configurations, *reducing the degree of discontinuity; whereas, for the condition number* measure, the switching happens instantaneously with few intermediate states, resulting in much larger values of joint velocity.

It is not clear right now why this difference exists. We will probably be able to get some clue, if the gradients of the two measure are first expressed in symbolic form and then each term is examined to pinpoint the cause of the abrupt switching. To have the symbolic expressions for this inspection appears to be possible, although quite complicated, for this particular manipulator case.

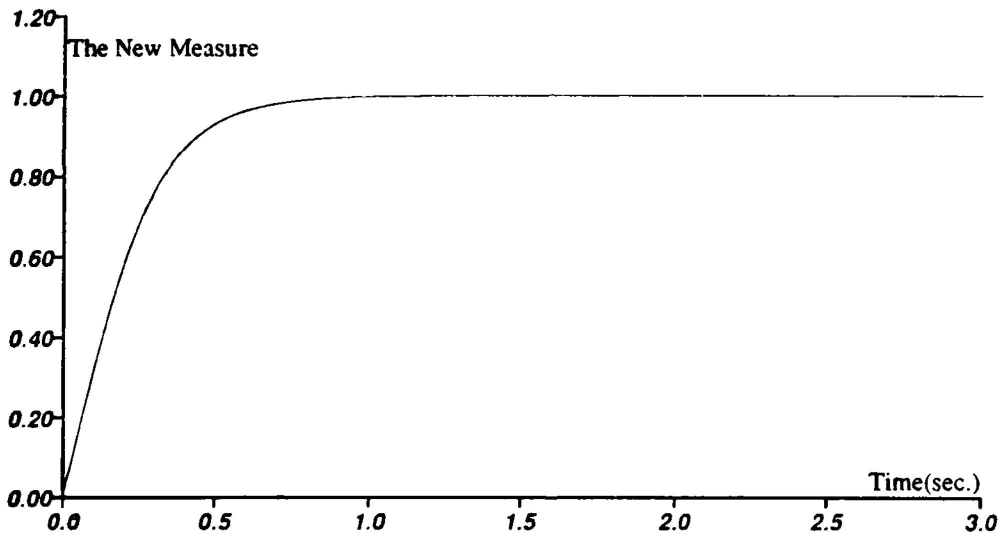
4.4 Conclusion

Summing up, the new measure has been compared with the two other measures, both qualitatively and quantitatively. The result of analysis on qualitative relationship agrees well with the experimental results.

To summarize these results, all the measures showed the ability to overcome singularity by successfully treating the two singular locations: the location corresponding to almost straight line configuration and the location with the tip at base or the origin. The essential difference between the new measure and the other ones is its ability to explicitly prevent the minors from becoming zero. This ability, in effect, prevents the merging of configurations and switching of aspects, which in turn prevents the repeatability problem and impulsive motions. In addition, balancing the values of minors appears to contribute to noticeably smooth movements near the base.

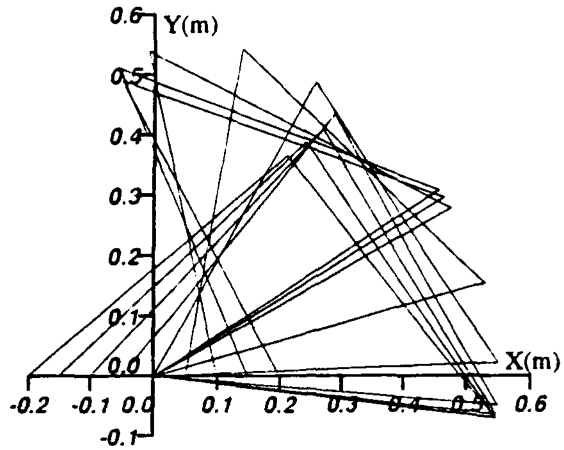


The New Measure Is Used

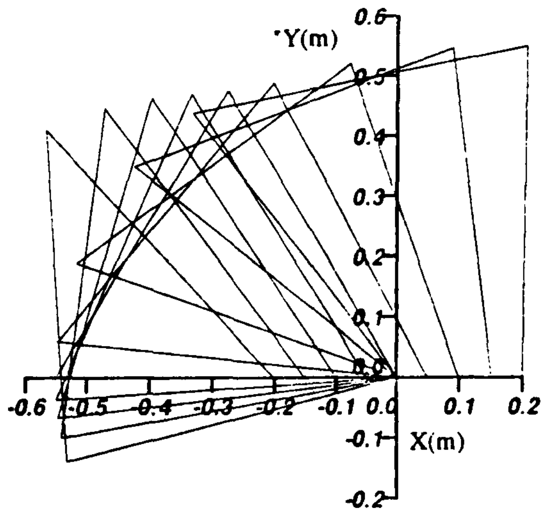


Time Response of The New Measure

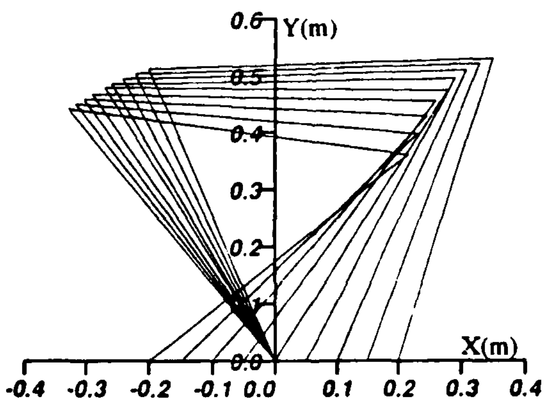
Figure 6: The Singularity Avoidance Ability When The New Measure Is Used: The Configuration Is Changing Toward The Optimal Configuration As Time Goes On. The Link Lengths Are $l_1 = l_2 = l_3 = 0.55m$.



The Condition Number Is Used

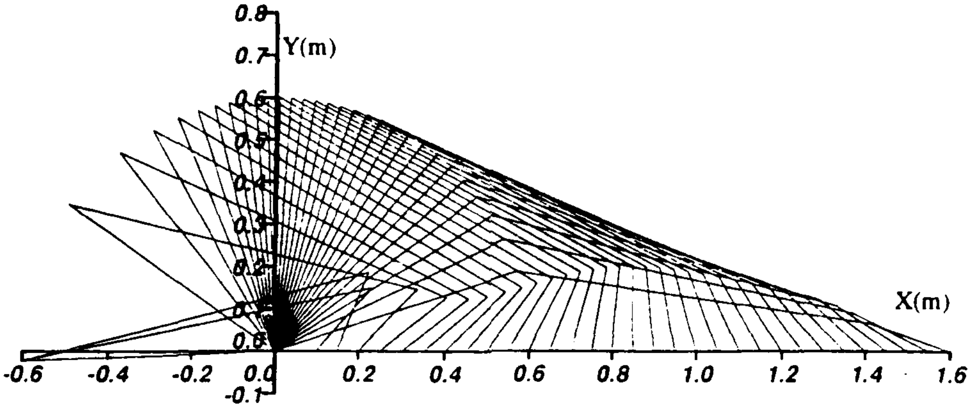


The Manipulability is Used

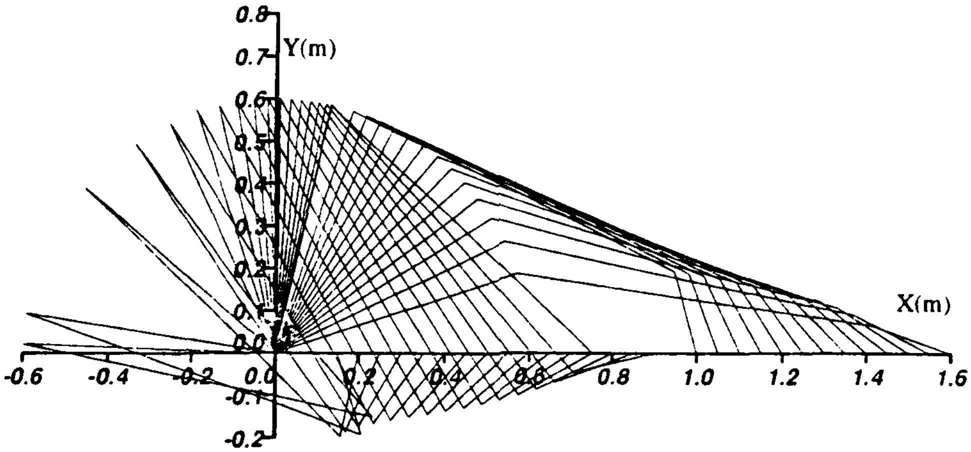


The New Measure Is Used

Figure 7: Use Of The Three Performance Measures, When The Tip is Passing The Base From $x = 0.2$ to $x = -0.2$ On The x-axis, With Link Lengths, $l_1 = l_2 = l_3 = 0.55m$.



The Manipulability Is Used In Configuration A



The manipulability Is Used In Configuration B

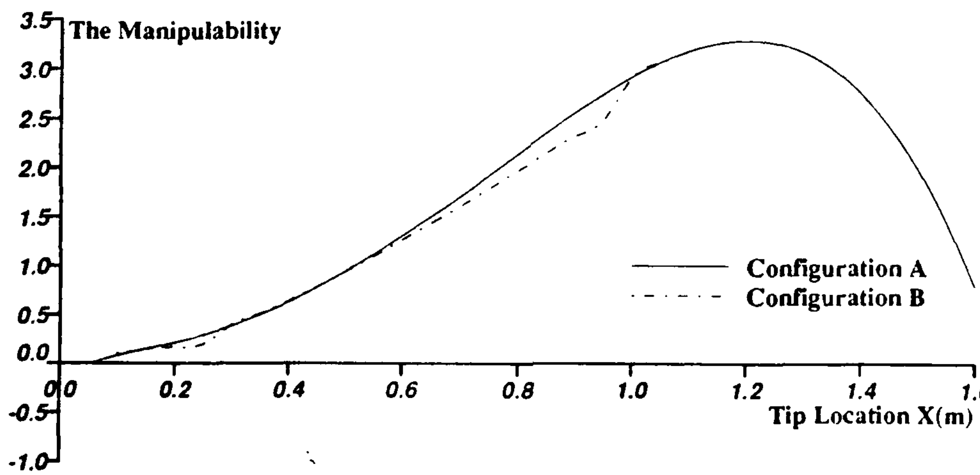
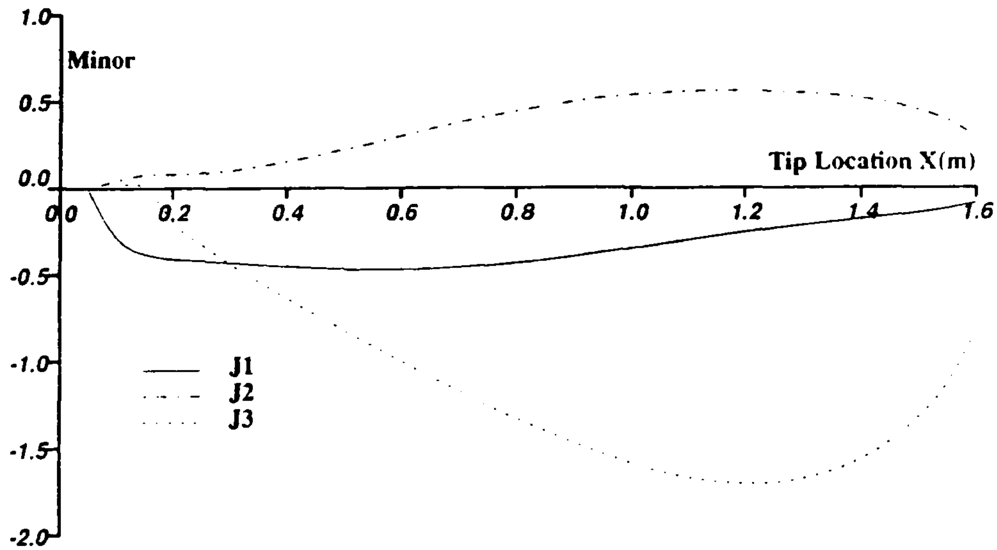
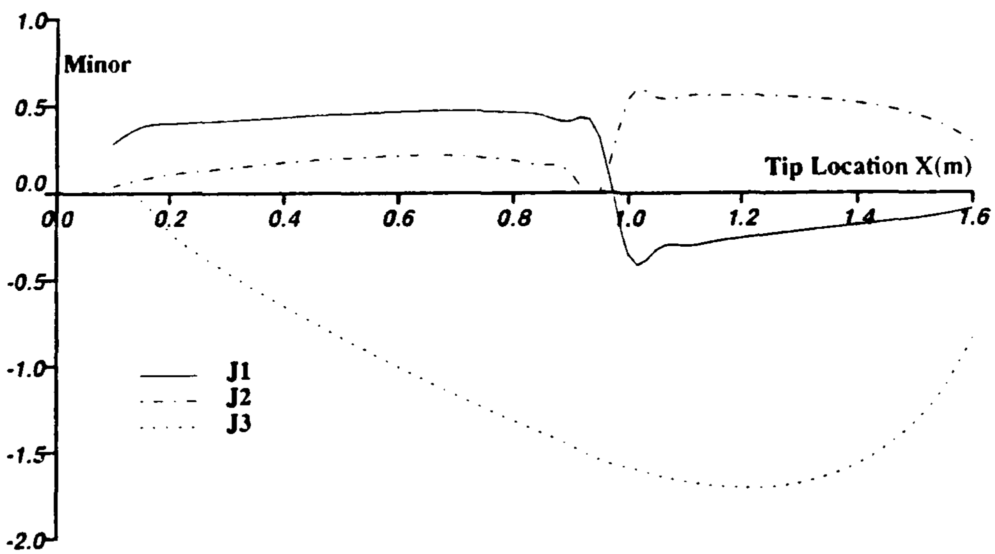


Figure 8: Configurations A And B, When The Manipulability is Used As Performance Measure, And Corresponding Values of The Measure, Where A And B Refer To The Tip-Motion With Different Initial Configurations.

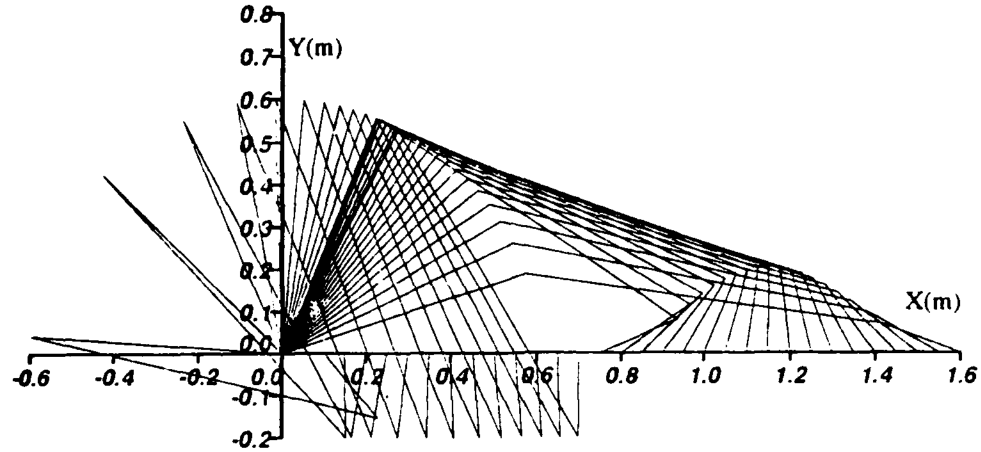


Minors For Configuration A

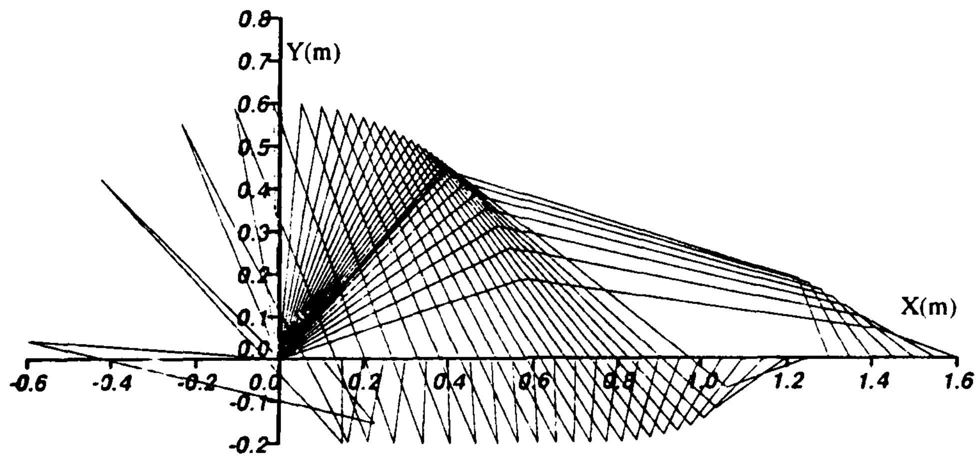


Minors For Configuration B

Figure 9: The Minor Values In Configurations A And B, When The Manipulability is Used As Performance Measure, Where A And B Refer To The Tip-Motion With Different Initial Configuration.



The Condition Number Is Used In Configuration A



The Condition Number Is Used In Configuration B

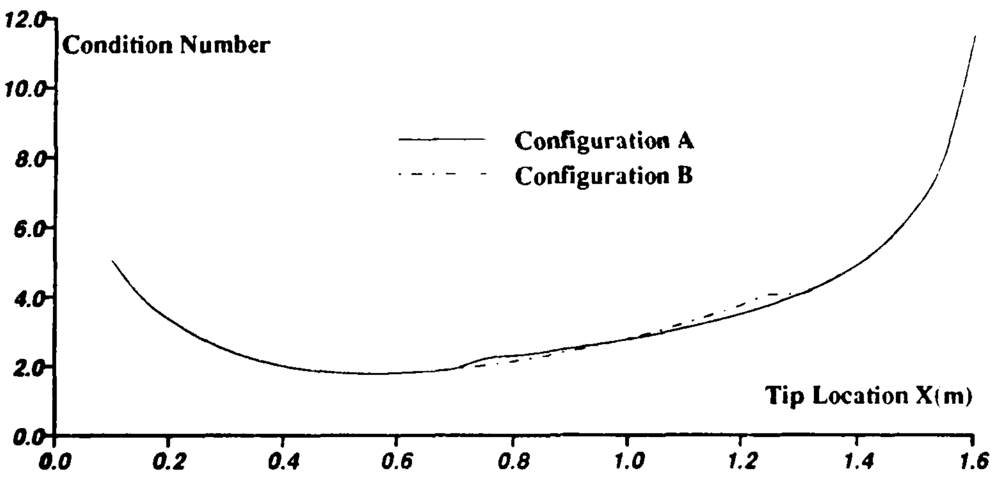
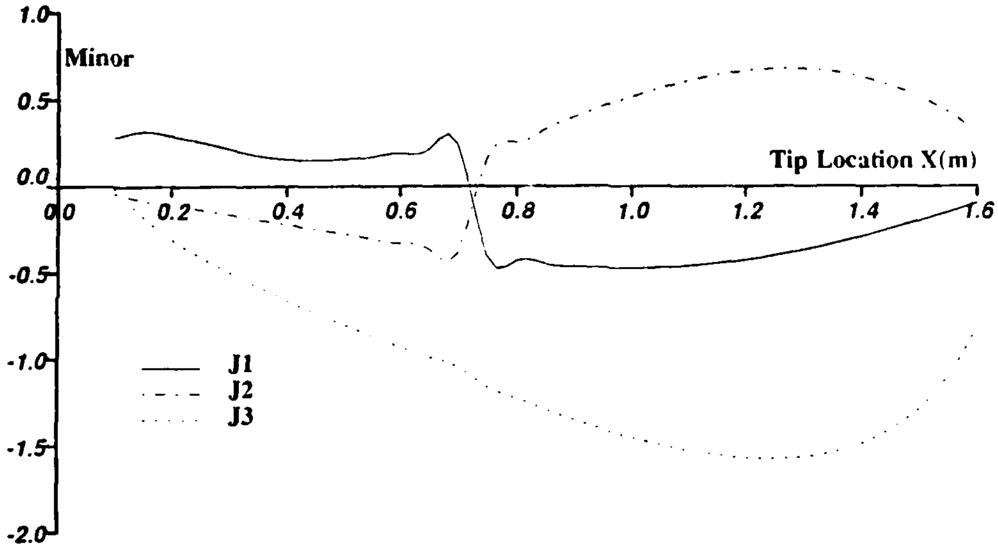
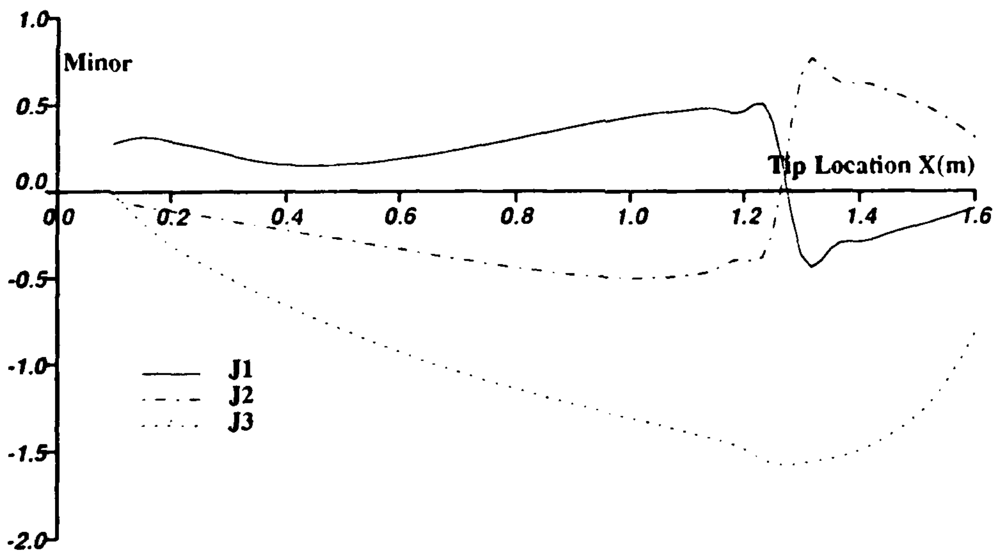


Figure 10: Configurations A And B, When The Condition Number Is Used As Performance Measure, And Corresponding Values of The Measure, Where A Refers To The Tip-Motion From 1.6 to 0.1 And B From 0.1 to 1.6.

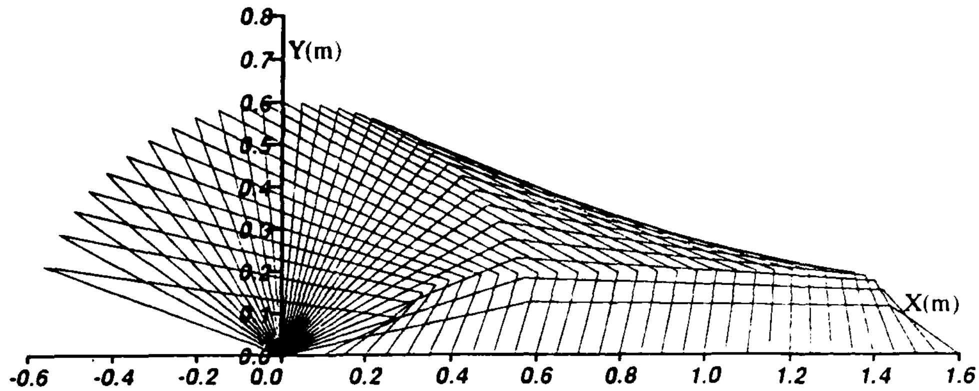


Minors For Configuration A

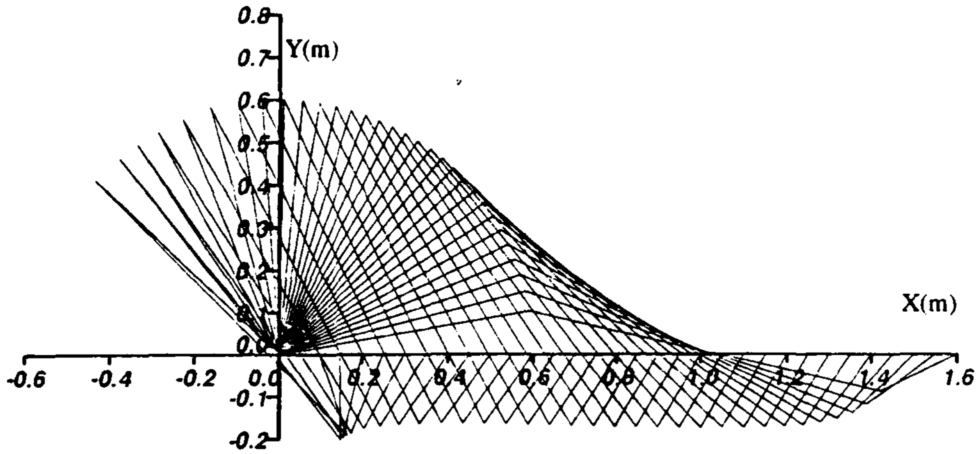


Minors For Configuration B

Figure 11: The Minor Values In Configurations A And B, When The Condition Number is Used As Performance Measure, Where A And B Refer To The Tip-Motion With Different Initial Tip Location.



The New Measure Is Used In Configuration A



The New Measure Is Used In Configuration B

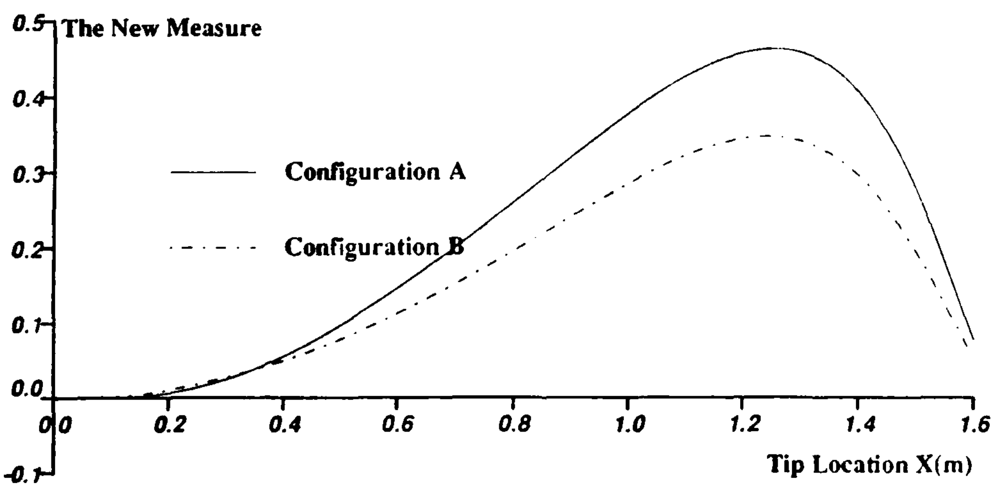
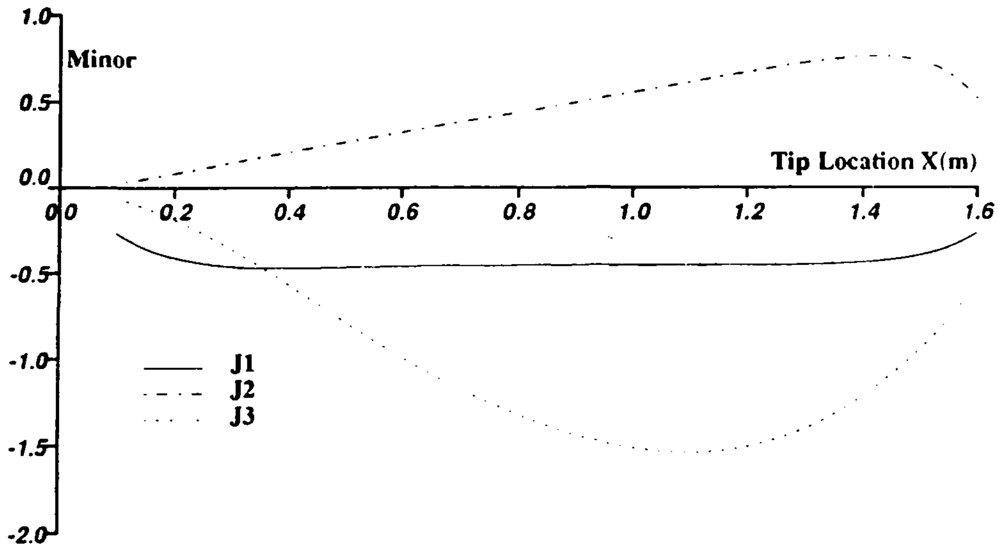
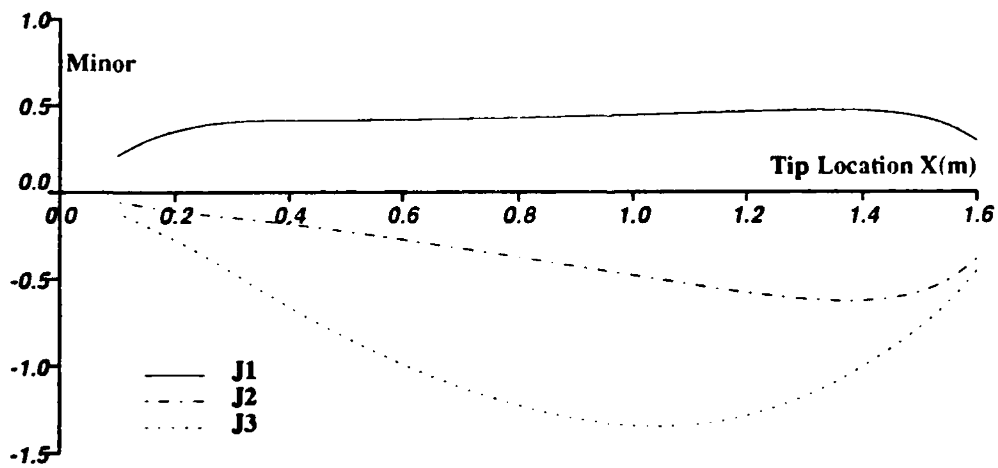


Figure 12: Configurations A And B, When The New Measure Is Used As Performance Measure, And Corresponding Values of The Measure, Where A And B Refer To The Tip-Motions With Different Initial Configurations.

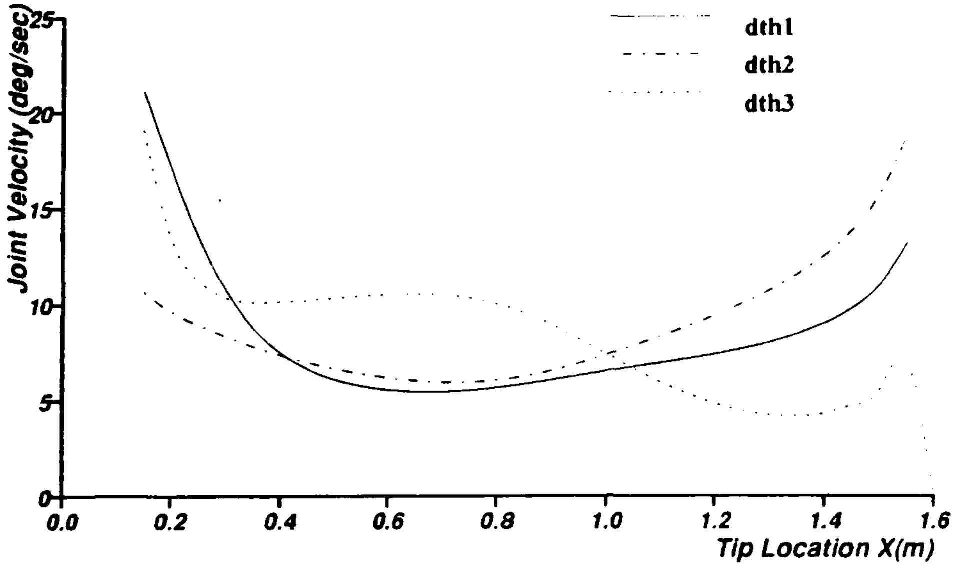


Minors For Configuration A

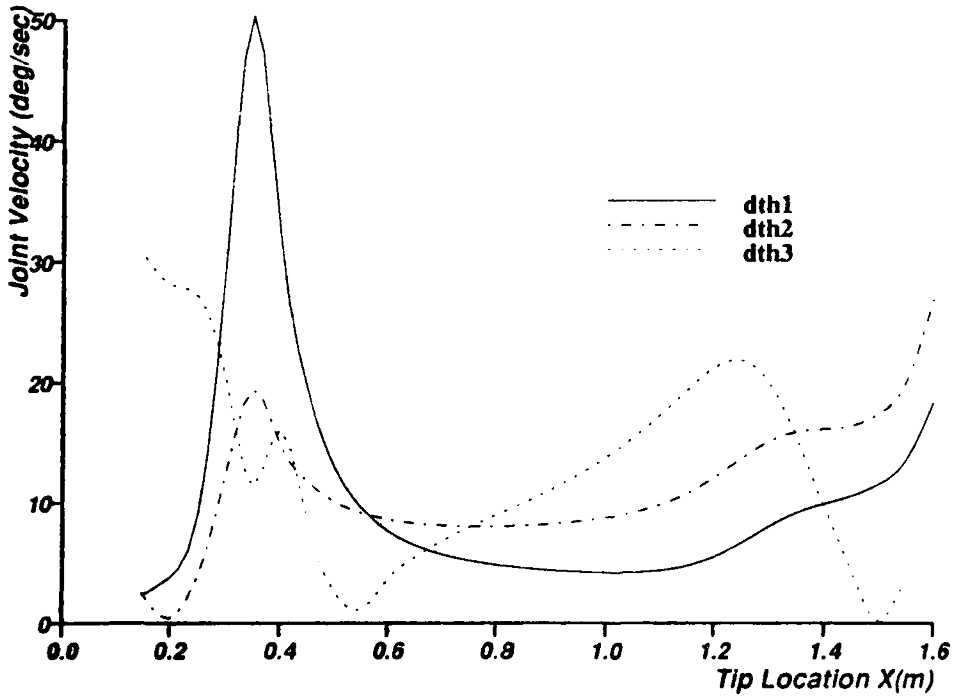


Minors For Configuration B

Figure 13: The Minor Values In Configurations A And B, When The New Measure is Used As Performance Measure, Where A And B Refer To The Tip-Motion With Different Initial Configuration.

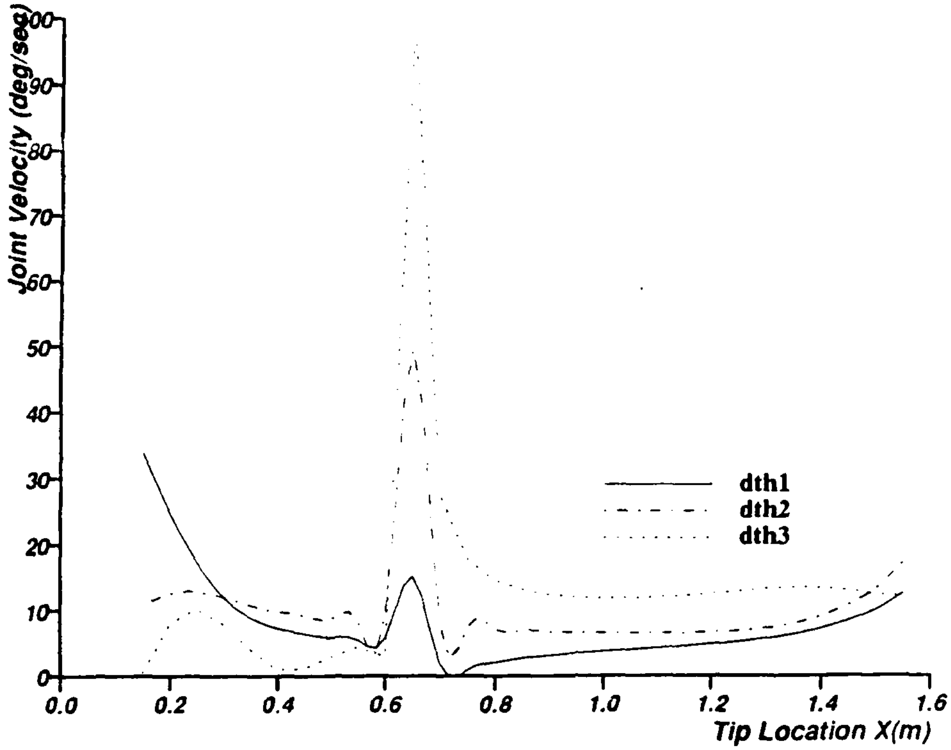


Joint Velocities With The Manipulability: Configuration A

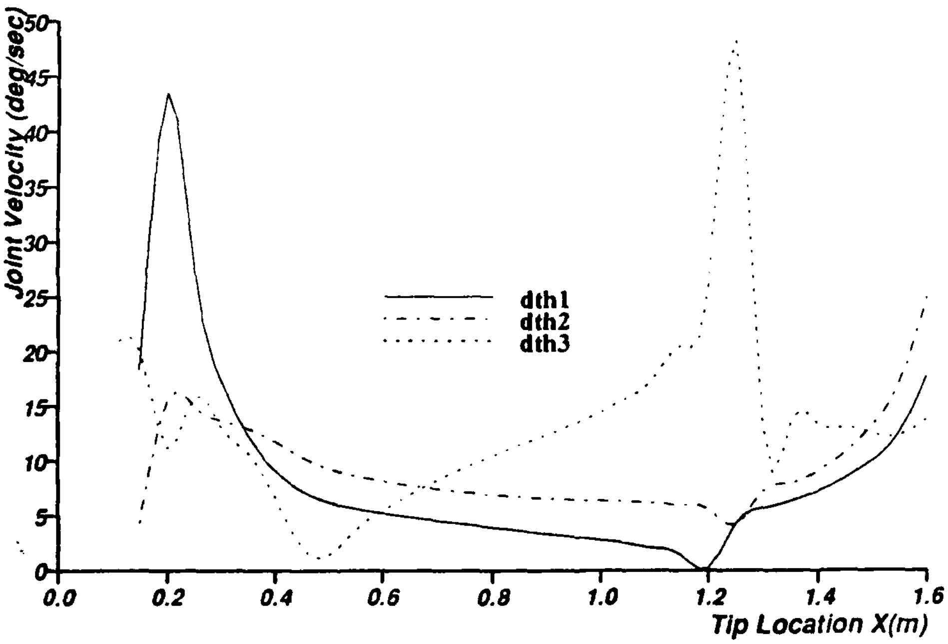


Joint Velocities With The Manipulability: Configuration B

Figure 14: The Joint Velocities In Configurations A And B, When The Manipulability is Used As Performance Measure, Where A And B Refer To The Tip-Motion With Different Initial Configurations.

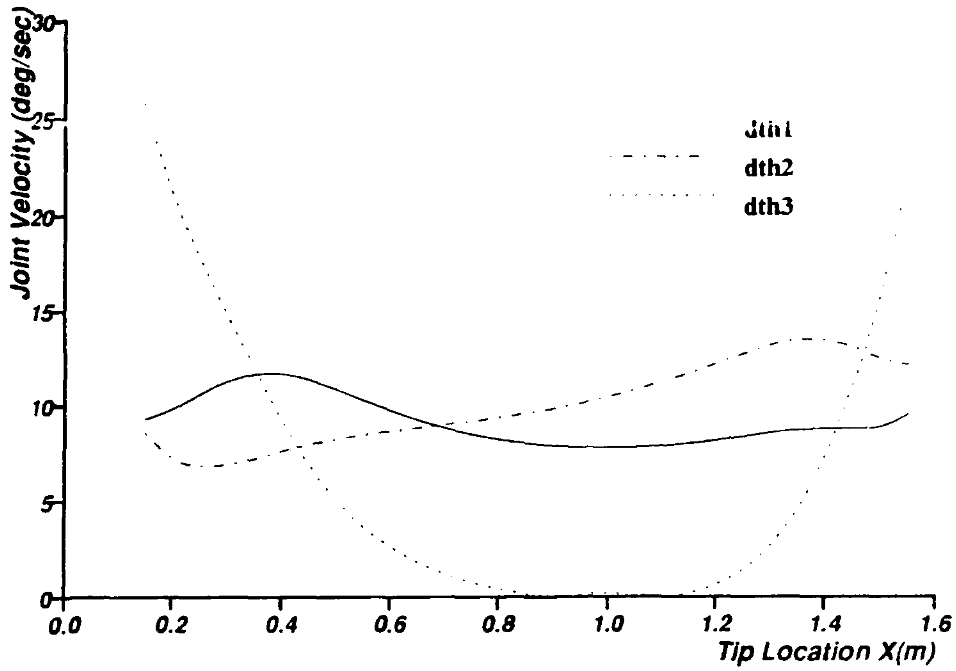


Joint Velocities With The Condition Number: Configuration A

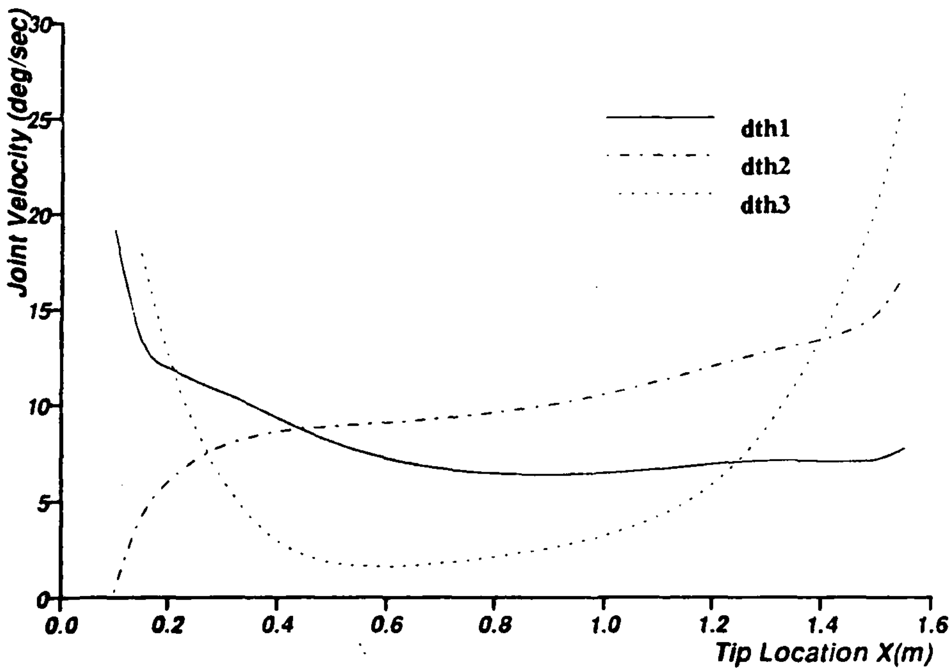


Joint Velocities With The Condition Number: Configuration B

Figure 15: The Joint Velocities In Configurations A And B, When The Condition Number is Used As Performance Measure, Where A And B Refer To The Tip-Motion With Different Initial Configurations.



Joint Velocities With The New Measure: Configuration A



Joint Velocities With The New Measure: Configuration B

Figure 16: The Joint Velocities In Configurations A And B, When The New Measure is Used As Performance Measure, Where A And B Refer To The Tip-Motion With Different Initial Configurations

5 Conclusion

In this paper, we have defined the concept of dexterity as a distance from singularity. Then we reviewed the concept of singularity and redundancy for further investigation of the distance concept. We have illustrated that there are different degrees of distance from singularity in the same degree of redundancy, showing that the conventional concept of redundancy is not sufficient to describe this distance. The new distance concept we derived was the number of nonzero minors as well as the magnitude of each minor. On the basis of the new concept, a new performance measure was derived. Then we have related the new performance measure with the manipulability measure and the condition number. Having investigated the qualitative relationship, we pointed out that the other measures do not have the ability to explicitly prevent minors from becoming zero. Through another series of numerical experiments, the effect of this ability was clearly confirmed. Whereas the two other measures without this ability showed repeatability problem and discontinuous motions, the new measure consistently overcame these problems.

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Appendix 1: Proof of Theorem 1

In this appendix, we make a proof of Theorem 1. The Jacobian matrix is expressed as the following:

$$\mathbf{J} = \begin{pmatrix} j_{11} & \cdots & j_{1n} \\ & \cdots & \\ j_{m1} & \cdots & j_{mn} \end{pmatrix}$$

Then

$$\mathbf{J}\mathbf{J}^T = \begin{pmatrix} \sum_{k=1}^{n} {j_{1k}}^2 & \cdots & \sum_{k=1}^{n} j_{1k} j_{mk} \\ & \cdots & \\ \sum_{k=1}^{n} j_{mk} j_{1k} & \cdots & \sum_{k=1}^{n} {j_{mk}}^2 \end{pmatrix}$$

In general, the determinant of an $m \times m$ matrix $\mathbf{A}$ is explicitly given as

$$det(\mathrm{A}) = \sum_{\sigma} (a_{1\alpha} a_{2\beta} \cdots a_{m\nu}) det(P_{\sigma}) \tag{1}$$

where a_{ij} is the element of $\mathbf{A}$ at i-th row and j-th column, $\sigma = (\alpha, \beta, \cdots, \nu)$ with distinct integers, $\alpha, \beta, \cdots, \nu \ = 1, 2, \cdots, m$, P_{σ} is the permutation matrix, and $\sum_{\sigma}$ means the sum is taken all $m!$ permutations of σ. Hence, the determinant of $\mathbf{J}\mathbf{J}^T$ is

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{\sigma} (\sum_{k=1}^{n} j_{1k} j_{\alpha k})(\sum_{k=1}^{n} j_{2k} j_{\beta_k}) \cdots (\sum_{k=1}^{n} j_{mk} j_{\nu k}) det(P_{\sigma})$$

Expanding this, we have

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{\sigma} (\sum_{k_1, \cdots, k_m = 1}^{n} j_{1k_1} j_{\alpha k_1} j_{2k_2} j_{\beta_{k_2}}, \cdots, j_{mk_m} j_{\nu k_m}) det(P_{\sigma}) \tag{2}$$

Here, note that terms that have non-distinct k_i's disappears. For instance, if $k_2 = k_1 = 1$, then $j_{11} j_{\alpha 1} j_{21} j_{\beta_1}, \cdots, j_{mm} j_{\nu m}$ disappears when $\alpha = 1$ and $\beta = 2$. Thus, in Equation 2, summation is applied only to the terms with distinct k_i's. Note also that the number, p, of different sets of distinct k_i's is

$$p = n\mathrm{C}m.$$

Rearranging Equation 2, we have

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{k_1,\cdots,k_m=1}^{n} (\sum_{\sigma} j_{1k} j_{2k}, \cdots, j_{mk} det(P_{\sigma}))(j_{\alpha k} j_{\beta_k}, \cdots, j_{mk} j_{\nu k})$$

Here summation $\sum_{k_1,\cdots,k_m=1}^{n}$ may be divided into

$$\sum_{k_1,\cdots,k_m=1}^{n} = \sum_{i=1}^{p} \sum_{\sigma_i}$$

where $\sigma_i = (k_1, k_2, \cdots, k_m)$ is i-th set of p different sets consisting of m distinct k_i's. Therefore Equation 2 becomes

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{i=1}^{p} \sum_{\sigma_i} (\sum_{\sigma} j_{\alpha k_1} j_{\beta_{k_2}}, \cdots, j_{\nu k_m} det(P_{\sigma})) j_{1k_1} j_{2k_2}, \cdots, j_{mk_m} \tag{3}$$

If we denote a part of Equation 3 as

$$\Delta_i = \sum_{\sigma} j_{\alpha k_1} j_{\beta_{k_2}}, \cdots, j_{\nu k_m} det(P_{\sigma})$$

Comparing this with Equation 1 shows that Δ_i is the determinant of the transpose of the submatrix made of k_i's column vectors as

$$\Delta_i = det([J^{k_1} \;\; J^{k_2} \cdots J^{k_m}]^T)$$

where J^{k_i} is the k_i-th column vector of the Jacobian matrix. Once a set of k_i's is chosen, the absolute value of Δ_i is fixed; only its sign changes as k_i's make permutations. If we set the absolute value as $|\Delta_i|$, Equation 3 becomes

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{i=1}^{p} (\pm) |\Delta_i| (\sum_{\sigma_i} j_{1k_1} j_{2k_2}, \cdots, j_{mk_m} det(P_{\sigma_i}))$$

The facts that the determinant of a matrix is equal to that of transpose of the matrix and that $\mathbf{J}\mathbf{J}^T$ is positive definite imply that

$$det(\mathbf{J}\mathbf{J}^T) = \sum_{i=1}^{p} \Delta_i{}^2.$$

Q.E.D.

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